# Dynamics of Scale*†

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**Abstract**

We develop the logic of 'scale' and apply it to the study of the general form of dynamical models of interacting bodies. We discuss the logical properties of space, time and motion, treating, in these terms, the boundaries of models, the relation between observer and observed, and the boundaries of the universe as a whole. We present a comprehensive philosophical description of the forces and constants fundamental to the physics of the world.

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# 1 Elements of Scale

The logic of scale is the logic of the concepts ‘large’ and ‘small’. It is the logic of ‘significance’, in that what is large in a physical system is what is important to its dynamics and what is small is what is to be ignored.[3] As a model of a physical system includes as constituent elements precisely those which are significant, so is the logic of scale the logic of the very existence of those elements. This logic is comprehensive, for a logic of existence is a logic of everything, insofar as „Sein ist offenbar kein reales Prädikat, [...]“[2].

## 1.1 Space and Time

Measurement in space is comparative—it involves the laying up of one object against another (the unit). The number of dimensions of a space is equal to the number of independent measurements taking place therein. Indeed, measurement in one dimension cannot epistemologically take the place of measurement in a second. This independence of spatial dimensions may be termed their ‘non-linearity’; a fixed (i.e. known) ratio of quantities is ‘linear’. Space, *qua* a multiplicity of dimensions, is generally the possibility for non-linearity itself. Space has three dimensions, so measurement in space is not actually binary but rather ternary: a unit measure must be composed of many independent objects; a metre-stick is a metre-stick as it is a copy of other metre-sticks.[1] If spatial measurement were binary, then a growth in the length of the object being measured, e.g., would be indistinguishable from a reduction in the length of the unit. When an object is compared to one of many copies, however, a change in the result of the measurement is unambiguous. Measurement in space is not quaternary, or of an even higher order, because then a single object would have two (or more) sizes. The unit of spatial measurement is unalterable, but it is in every other way indistinguishable from the object that it is used to measure. Spatial measurement, and space itself, as it were, are undirected: the measurement of the length of one object in terms of the length of another is syntactically identical to the measurement of the length of the latter in terms of the length of the former; if someone says “We are three klicks from the water tower.”, he could either be stating a fact or yet he could be defining ‘klicks’. A directed comparison, on the other hand, is precisely one which identifies the unit absolutely. Thus, the undirectedness of space may also be called its continuity, or its [infinite] divisibility, for continuity is nothing but the absence of an absolute unit.

Time is the opposite of space, small-scale instead of large-scale, quantised instead of continuous. Indeed, time can be divided only into [indivisible] ‘moments’: while one may always fold a spatial unit in half to measure a smaller distance, there is no similar act by which a clock may be sped up.[2] If the arm of a pendulum is shortened, for example, there is no way of knowing that all of the other components of the clock remain unchanged; temporal measurement is

[1]The three elements of spatial measurement are (1) the object being measured, (2) the part of the unit being employed directly, and (3) the rest of the unit, which may verify the integrity of the part in use.

[2]This edict does not hold only for an Einsteinian ‘light clock’, because of such a clock’s relativistic nature. A continuous clock may be constructed precisely at the largest scale, where time, by the Law of Diminishing Returns, becomes like space (see Section 2.2).

not ternary but unitary. Time, of course, is directed, as space is not, for, while every bit of matter is different from all others, moments pass in the counting of [indistinguishable] repetitions of an event. Accordingly, time is linearity instead of non-linearity.

Each element of a model is a dimension of that model in phase space, so any increase in the number of elements that a model describes is the introduction of a non-linear relationship into that model's dynamics; a decrease in the complexity of the model, correspondingly, is a reduction in the number of non-linear relationships. The number of elements of a model may change either reversibly (spatially) or irreversibly (temporally). If it changes reversibly, then particles are added to and subtracted from the system only by the division and combination of other particles, as each of these processes is undone by the other. Every reversible reduction in the number of non-linearities is an increase in the number of linearities; every such increase is a decrease in the number of linearities. Two variables may be approximated as one variable precisely when those two variables are related by a linearity. If a change in a system is irreversible—directed—then that change may be universally represented as involving 'accumulation'. Indeed, time adheres to the Law of Diminishing Returns, which [uniquely] describes all accumulative processes and their irreversibility.

## 1.2 Global and Local Motion

Global motion is motion of parts of a body relative to each other. Local motion is motion of a body as a whole [relative to other whole bodies]. A prototypical example of global motion is rotation; of local motion, translation. Global motion is large-scale local motion, and local motion is small-scale global motion.[3] All motion is defined only relative to a given background: local motion is defined relative to a nearby background, while global motion requires the presence of a background that is distant. In other words, a man may fly toward the heavens, but the heavens may not fly toward a man, while a man may spin on the ground, but the ground may not spin on a man. On the other hand, the heavens may spin around a man, and the ground may fly toward a man.

Identically-constructed objects in relative motion are indistinguishable precisely if they are near to each other and move locally or if they are far apart and move globally. With two nearby objects, the motion of one to the left is equivalent to the motion of the other to the right, though such objects in relative rotational motion are not identical. If, however, the distant background is occupied by a copy of an object in the foreground, then the rotation of one of these bodies involves the counter-rotation of the other and the two are indeed indistinguishable, yet in this circumstance translation is not relative.

The 'nearby' background of an object includes precisely that which is a [finite] distance away, or 'external'. The 'distant' background includes that which is infinitely distant, or 'internal'. (Neither the infinitely close nor the infinitely far away may be reached by local motion, while both of them are involved in global motion.) One may speak of the distinction between the external and the internal in terms of the occupation of space: the vacuum may be contrasted with the bodies that it separates—the difference between matter and void is a difference of scale. The nearby and the distant backgrounds are

[3] Rotation through a small angle involves great translation [in faraway objects], and so on.

not independent, however, as matter may move from a body's interior to that body's exterior or *vice versa*, and as an object in space may grow (shrink) and thereby cause the nearby background to shrink (grow). Indeed, the size of a single body is defined relative to the sizes of other bodies, but the size (mass) of all bodies is determined relative to the size of the nearby (distant) background (see Sections 3.2 and 3.3).

## 1.3 Ranged and Contact Forces

The union of space and time is the possibility for motion—for directed non-linearity. Motion is division and combination *plus* direction; it is not only the creation and destruction of particles, but the [non-linear] feedback (positive or negative) of the force which precipitates it. That is, motion is *self*-promotion and *self*-destruction, the recursion of which is a manifestation of motion's directedness: time—linearity—is the scaling of a system [in the absence of feedback] and *scaling must itself scale.* Force is just another kind of motion, as all motion is relative and as all frames of reference are equally valid.[4] There are two kinds of forces, 'ranged forces' and 'contact forces', and there are two 'aspects' to each kind of force, these aspects corresponding to motion's two directions, forward and backward. The strengths of the aspects are related non-linearly, and whichever aspect is stronger, the feedback of the force points in that direction, its magnitude equal to the difference in the magnitudes of the aspects. If the force in question acts at range, then the two aspects are attraction and repulsion; otherwise, they are propulsion and collision. In general, the two aspects of any force are representable by division and combination, and the feedback associated with one aspect of a force is opposite that associated with the other. Attraction, repulsion, propulsion and collision are all non-linear, as division and combination are non-linear, division and combination being reversible and spatial.

The magnitude of a ranged force is based on a comparison of the respective distance-dependencies of its attractive and repulsive aspects. Because this magnitude must decrease as the distance from the source increases,[5] attraction, which diminishes that quantity, has positive feedback, while repulsion, which does the reverse, is self-destructive. All contact forces must be dependent only on the masses of the bodies involved [and not [also] on distance]. By 'ejection' and 'injection', matter leaves and enters a body, adding to and subtracting from the nearby background, respectively. The feedback of these processes lies in the fact that the more matter is expelled from a body, the greater is the acceleration caused by further propulsion [with the same propellant], while the more matter there is that has been collided, the less is the significance of further [otherwise identical] collision, because the already-collided matter, too, must then be propelled.

The action of every aspect of every force is non-linear, as any change in the size of a body is non-linear, and as space is three-dimensional (along with the bodies that occupy it). Precisely with three dimensions of space does every body have associated with it exactly one quantity that represents the non-linearity

[4] Acceleration is motion of velocity, jerk is motion of acceleration, and so forth.

[5] It is when bodies are near to each other that small changes in their respective positions are significant, so ranged forces must be great precisely when the bodies that they act on are nearby.

of that body as a whole, which quantity must be both a comparison of other [independent] quantities (space and non-linearity themselves being comparative) and equal to all other candidates [up to a linearity]. These conditions are met in three dimensions by the ratio of surface area to volume: with every increase or decrease in this value, the rate of that change itself changes, manifesting the feedback inherent in the process of the growing or the shrinking of the body; in the action of the forces that it feels. No such quantity exists in two dimensions because the perimeter and area of a two-dimensional figure are not independent and because there is no such thing as, for example, the width of an irregular pentagon. With more than three dimensions, on the other hand, there are multiple, inequivalent non-linear quantities, and so no one possibility suffices.

# 2 Boundaries of Scale

The boundaries of a model, the limits beyond which that model is not valid, may be understood in terms of the scales beyond which the model does not apply. That is, the boundaries of a model specify the scope of the linearity of the physics that it describes; it is precisely outside these boundaries that non-linear effects are present. The non-linearity above the maximum scale is self-promoting, and the non-linearity below the minimum scale is self-destructive.

## 2.1 Maximum and Minimum Speeds

As there are boundaries to every model, so are there limits to these limits—limits to [the boundaries of] all possible models. But as all models are models *of* the universe, the boundaries of a particular model and the boundaries of all models are not the same sort of thing. The latter, specifically, restrict the relativity of motion as universal minimum and maximum speeds that elements of any model may attain. A speed defines a ratio of distance in space to duration in time, so, as space is non-linearity and time is directedness, extremal speeds are extremes in the magnitudes of dynamical forces. These boundaries, as universal, are valid in every frame of reference. Nevertheless, their existence signifies that motion is not completely relative (indeed, that it is non-linear): motion at the extremal speeds is qualitatively different from motion of moderate magnitude. As space itself is infinite, and as every observer has his own distant background, so the universe as a whole is infinitely 'deep'. However, no model may be infinitely deep.[6] Rather, models are bounded by a maximum depth—the speed of light in a vacuum (denoted $c$). The minimum speed, the orbital speed of the Bohr electron, which we here label '$d$', prevents the collapse of atoms, allowing that collections of particles be able to act as bodies and forbidding that there exist any matter of infinite 'density'.

The minimum speed applies only to particles, and the speed of light applies only to bodies. Accordingly, small-scale elements may travel faster than $c$, and macroscopic objects may travel slower than $d$. The minimum speed is, however, a minimum speed for the transmission of light through media, and the maximum speed is a maximum speed for elementary particles in a vacuum; $d$ treats the transmission of waves, and $c$ treats the transmission of particles. Only insofar

[6] This is the logical basis for Olbers' Paradox.

as wave packets act like [virtual] particles do particles [propagating in empty space] behave like waves, and this only happens at the extremal speeds, as that is where global and local motion meet, as it were. Indeed, motion is wave-like at small scales; is particular at large scales.

The speed of light, limiting the rate of communication in a vacuum, specifies what it means for something to be 'distant'; the speed of the Bohr electron, limiting the rate of communication in the interior of a body, specifies what it means for something to be 'nearby'. As each extremal rate of communication is directly and bijectively associated with a location, so does it also determine the quality of observation that takes place there. In quantum physics, the act of observation is self-destructive: [precisely in the act of observation] the observer destroys his own ability to observe [that same system again].[3] Observation of distant objects is self-promoting, for such objects recede from the observer as they are observed (with the action of gravity: see Section 3.1) and so, in the process, become continually harder to disturb. In between the extremes, in the linear regime, the observer may observe either actively (disturbing the observed system) or passively (without interference), as he wills. With the minimum speed limit arises the Uncertainty Principle; with a maximal speed limit comes the light cone.

## 2.2 Space-Time and Architecture

Beyond the universal boundaries, the division between large and small—between space and time—breaks down. As all linear processes are non-linear at large scales, when a system is old, increments of time, by the Law of Diminishing Returns, become insignificant next to the age of the system. With temporal quanta small, time is continuous and undirected like space, and there are four spatial dimensions, this quadruple termed 'space-time'. At the smallest scale, on the other hand, space becomes quantised and directed like time. Indeed, at this scale, all values are quantised—there exist 'natural units', by which measurements of all sorts are determined. Because of this, space cannot there be continuous; instead it is directed, e.g. across atomic orbitals. The three dimensions of space, necessary for dynamics between the extremal scales, collapse into one dimension in the quantum realm, where all physics takes place in two dimensions of time, which dimensions we call 'architecture'.

The curvature of space-time is determined by the distant background, while the structure of architecture is determined by the nearby background. The distant background for small-scale dynamics is rigid and unmoving, and the nearby background for physics at the largest scale is simply empty (and so, too, undynamical). Architecture is a description of the interiors of bodies, unlike space-time, which treats the (external) void; there is no space-time inside bodies, and there is no architecture among them. Thus, space-time is the motion of the distant and describes the nearby, while with architecture it is the opposite. The quantisation of energy levels is founded on the existence of a minimum speed, as the possibility for curvature of space-time depends on the the speed of light.

Where space becomes like time, motion becomes quantised and pseudo-rotational (see Appendix A). Where time becomes like space, motion both forward and backward in time is possible, the former with the time-dilation of Special Relativity and the latter with pseudo-translational motion through wormholes. Indeed, the speed of light accounts for the existence of black holes and white holes; for the edges of space-time. In contrast to Quantum Mechanical spin, which discretises architecture, wormholes make space-time continuous even at event horizons.

## 2.3 Determinism and Chaos

Time is the development of systems by random changes of state, as it is random dynamics that relate moments of time. Microstates, too, have random dynamics, and a system with microstates is deterministic,[3] so any temporal process deterministic, and *vice versa*. Indeed, determinism is directedness, and directedness is time. Chaos, the opposite of determinism,[3] is spatial, for non-linearity is nothing but lack of predictability (i.e. lack of determinism) [in measurements]. Chaotic motion is the motion of a system through space; it is local, rather than global (and deterministic), motion. Thus, internal motion, and the motion of the distant background, is deterministic, temporal and linear, while external motion, the motion in the nearby background, is chaotic. (Non-linearities are outside of the model; linearities are inside.)

The number of dimensions of a system indicates whether or not observation in that system is active (and the dynamics are chaotic) or passive (deterministic). This is clear from considerations of the interaction between an observer and a two-body problem that he observes. If the space in which this occurs is two-dimensional, then the observer must himself be a part of the system which he is observing, a system which is then a three-body problem and chaotic: any motion of the observer's that is sensitive to changes in the relative position of the binary system will, by the relativity of motion, alter the system just by being so (see Figure 1). In three dimensions, where the observer is capable of motion along the line which is perpendicular to the plane of the observed two-body problem and which intersects its centre of mass, he is able to detect relative motion in the pair of bodies without affecting them in the process, though he may yet disturb their motion if he moves otherwise. With four-dimensional space, the observer cannot interact with the system: he must move only in the plane perpendicular to the observed system and intersecting its centre of mass. If he did not, then he would form part of a two-body problem in which the other body is, impossibly, the three-body problem of his observation.

All two-dimensional physics is the physics of architecture, of the nearby and of chaos; four-dimensional physics is the physics of space-time, which is perpetually distant and therefore deterministic. In general, large-scale physics are deterministic, while small-scale physics are chaotic. Large-scale objects—black holes—are great conglomerations and necessarily rare, their dynamics modelled by the two-body problem; elementary particles, in contrast, have few constitutive parts and are by their nature numerous (their interactions described by many-body problems). Black holes tend to combine, producing fewer, larger bodies, and with self-promotion, while elementary particles tend to divide, producing more elementary particles, becoming, self-destructively, ever more particulate. Indeed, division itself is chaotic and combination itself is deterministic—the end of a series of combinations is definite (knowable; predictable), whereas there is no [definite] end to division.

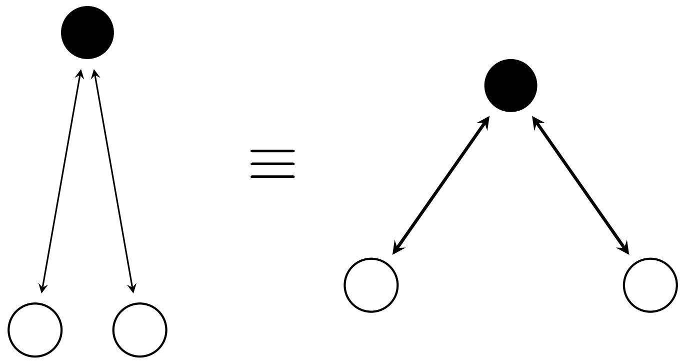

Figure 1: Large-scale changes in the configuration of the general $n$-body problem have no necessary small-scale effects, and there are short-term perturbations with long-term consequences. The removal of the ambiguity in the system's state, such as is necessary in an act of observation, is effected by the motion of the observing element.

# 3 Physics of Scale

The logic of scale describes the general form of physical laws, if not solutions to engineering problems or results of scientific observation. Indeed, the scale of the physics in question determines both the qualities of the forces involved and the nature of the physical quantities that determine their magnitudes. All of the physics of a given system thus follow from the scale of the system, and physics itself may be exposited entirely in terms of the logic of scale.

## 3.1 Gravity and the Quantum Force

Exactly one force acts in models whose boundaries are near the maximal boundary of the universe itself. This is gravity, and its nature is made clear by a re-examination of Einstein's rubber sheet model, the circularity of which indeed suggests that gravitational attraction is a necessary feature of the dynamics of space-time. By the Equivalence Principle, the force induced on the bowling balls is understandable as arising from upwards acceleration of the supporting rubber sheet, which will stretch in dragging the weights along with it. Translating this picture into four dimensions, the motion of the rubber sheet becomes the expansion of the distant background, which expansion is visible in the curvature of space-time: in any frame of reference in which the distant background is expanding, all nearby objects must accede (see Section 1.2).[7] The repulsive aspect of gravity corresponds to the other way in which space-time may stretch, i.e. by the contraction of a rotating distant background. This repulsive force is the centrifugal force, and, in fact, attractive gravity and centrifugal acceleration are opposites in every way except in magnitude.[8] Gravity is the force of

[7]The only difference between the model of the rubber sheet and that of the curvature of space-time is that a stretching of the rubber is a contracting of space-time, and *vice versa*. However, this apparent inconsistency arises only as a consequence of the fact that the former model is embedded in a higher-dimensional space while the latter is not.

[8]In the attractive aspect of gravity, there are laterally compressive forces; in the repulsive aspect, laterally dispersive ones. The tidal forces of the two aspects are also in opposing

the largest scale, so it is nothing but the non-linear effects of the composition of matter, inheriting the positive feedback built into the logic of attraction ('gravity gravitates').

The force of the smallest scale, 'the quantum force', manifests the dynamics of architecture. Its 'propulsion' produces virtual particles and radioactive decay; the collision of elementary particles is the binding of those elements together, sometimes, perhaps, with their mutual annihilation. A general picture of changes in architecture may be formed in consideration of the Casimir effect. The parallel plates which demonstrate this phenomenon, by their proximity, provide a nearby background against which small-scale phenomena may be measured. It is the quantum force, which works ultimately for division, that draws the plates together and does so with self-destruction until the minimum scale is reached (gravity is self-promoting until space-time itself breaks down, as occurs with the formation of singularities). The plates are drawn *in*, and this is equivalent to the drawing of the particles themselves *out*.

There is a third fundamental force in the realm between the large and the small—the force of electrostatics. In contrast to gravity and the quantum force, electrostatics has force laws of attraction (collision) and repulsion (propulsion) that are of the same dependence on distance (of the same magnitude of contact force): electrostatics has no favoured direction—no preference for pulling or pushing, injecting or ejecting. (Indeed, electrostatics may act either by contact or at range.) Electrostatics is identical to all linearity in motion: electrostatic fields do not interact either with themselves or with each other (this is the principle of superposition). Consequently, there is uncertainty in the number of fields that are part of the description of any given electrostatic system. This uncertainty may be stated in terms of the equivalence of all regions of zero electric potential, and the presence of this uncertainty is the condition for the applicability of the 'method of images', from which it follows that electrostatic attraction and repulsion (propulsion and collision) are able to counteract each other [completely], as is impossible with the other two fundamental forces.[9]

## 3.2 Charge and Mass

Charge is the measure of [spatial] extent; it is equal to the number of elements in a system, this the quantity which is altered by division and combination. Mass is the measure of duration; it is that which is accumulated with the passage of time. Mass determines the strength of contact forces, while charge determines the strength of ranged forces (see Section 1.3)—indeed, contact forces act at the boundaries of bodies, which are temporal, while ranged forces operate over expanses of space. Charge, i.e. volume, is measured by the (continuous) displacement of fluids. Measuring the mass of an object, on the other hand, involves balancing it with iteratively-generated accumulations of bodies of known weight.

There is one charge at the largest scale, as gravity is singularly attractive, and there are many charges at the smallest scale, as the quantum force is divisive. There is precisely one mass between the extremal scales, but there are two

directions, if, as is proper, one considers only differential rotation with conserved angular momentum.

[9] This is not to say that there are no frames of reference in which there is no gravity or no quantum force.

masses at the largest scale, for the charge of the largest scale is itself a mass. These two masses, the inertial mass and the gravitational mass, are not identical, precisely insofar as gravity is non-linear. Likewise, mass in quantum physics is just one of many charges.

The division of electric charge into two [opposite] polarities follows from the linearity of electrostatics, for it is precisely the dynamics of such charges that are balanced between determinism and chaos; between division and combination. In the observation of a pair of interacting electric charges, there are two cases to be considered: either the observer detects attraction [in two bodies of opposite charge], or he observes repulsion [in like-charged elements]. In the former case, whatever the charge of the observer himself, the system as a whole is a two-body problem, by the possibility for electrostatic shielding. In the latter case, any change in the arrangement of the observed elements changes the strength [and possibly the polarity] of the force that the observer feels, so the system is a three-body problem and of chaotic dynamics.

## 3.3 Large and Small Numbers

The magnitudes of the non-linear fundamental forces change over time and over space. The difference of their strengths is what one usually calls 'cosmological time' ($\tau$), and it is equal to the age of the universe. This quantity is to be contrasted with the *ratio* of the universal boundaries, the 'fine-structure constant' ($\alpha = c/d$). Instead of a duration, the fine-structure constant is an extent; it is a comparison of extremal distances (durations) [to be travelled (spent) in a given period of time (travelling over a given extent of space)].

Cosmological time, as an age, is directed, and the fine-structure constant is undirected. The directedness of $\tau$ and the undirectedness of $\alpha$ are apparent in their great and small values, respectively: where a small number is similar to all of its inverses, a large one is the opposite. Indeed, one can speak of a large number as having 'made a commitment'; as having inherent direction ('Up!'). Small numbers are more 'precise', and changes in their values are continuous. The fine-structure constant has a small value, since, with all [spatial] measurement, the unit and the object measured must be of approximately equal size. Large numbers, then, are discontinuous.

There can only be one large number, and this number must be equal to both the age and the matter content of the universe.[1] On the other hand, there are many small numbers, but they are all close to $\alpha$. Any number between $\alpha$ and $\tau$ is a value of a particular model and not one of the universe as a whole. The quantities of models are always to be contrasted with those of world: definitions of the boundaries of a model are meaningful only in terms of the boundaries of the universe [in space or in time]. Indeed, space is large-scale yet $\alpha$ is small, while time is small-scale and $\tau$ is large. Spatial magnitudes are measured in terms of $\alpha$, and temporal durations are fractions of $\tau$. The universe has a definite age and a definite size. So, too, does it have a beginning, but not a beginning in *time*, and it has a size, but not an extension in *space*.

## Appendix A Fermions and Bosons

Consider two elementary particles in relative 'rotation'. In the frame of one of the particles, the second particle is both orbiting the observer and spinning about an internal axis. There are exactly two possibilities: either the paired particles are identical, or they are not, in which case there is present an extra degree of freedom, namely that of reflexion. Differences in the motion of the second particle are undetectable by the first just as long as the period of the second particle's axial motion is a certain multiple of the period of its orbit: if the two particles are not identical, then the periods must be related by integer multiples; if the particles are identical, then their spins are related by half-integers, this possibility allowing that the second particle, rotating around the first, should present at the end of its orbit the 'face' opposite to that which it presented at the start. Coupled with the requirement that there exist a minimum angular velocity, this picture of the pseudo-rotation of the smallest scale shows the origin of quantisation in spin and of the categorisation of elementary particles either as fermions or as bosons.

## References

[1] P.A.M. Dirac. Cosmological Constants. *Nature*, 139(3512):323, 1937. doi:10.1038/139323a0.

[2] Immanuel Kant. *Kritik der reinen Vernunft*, page B627/A599. 1787/1781.

[3] Adam Krellenstein. Microstates, Macrostates, Determinism and Chaos. krellenstein.com/adam, June 2011. `urn:uuid:802e61fc-62a4-4aa2-be2c-d2b1289deed5`.