Microstates, Macrostates, Determinism and Chaos

Adam Krellenstein*

June 2011

Abstract

First, we show that the Second Law of Thermodynamics and Darwinian Evolution are founded on arguments of identical form, the employment of which is sufficient for drawing concrete conclusions as to the gross behaviour of diverse systems without making use of physics proper. Next, we assert, through the example of the n-body problem, that these arguments are a part of the description of any physical system which may be called deterministic; chaotic systems, in contrast, are unpredictable precisely because they cannot be thus treated. Finally, we explain some non-classical features of Quantum Mechanics as following from the impossibility of fulfilling the prerequisites of determinism at the level of elementary particles.

1 Introduction

Statistical and biological systems provide the starting point for this investigation into the logical foundations of physics, from which it follows that determinism may reduced to this principle: that when a system is able to be described at two different scales simultaneously, a probabilistic treatment of the assignment of small-scale states to large-scale states will specify the system's large-scale behaviour for arbitrarily far in the future. Prediction is then this act of specification and is possible for precisely those systems which may be described at different scales.

Adopting the terminology of Statistical Physics, henceforth the descriptions of a system at the smaller scale will be called 'microstates', while their larger counterparts will be called 'macrostates'. In developing a generalisation of

*Electronic address: adam@krellenstein.com

the Second Law of Thermodynamics, we attend the ratio of microstates and macrostates itself, instead of the logarithm of that value. We provide a probabilistic argument for the maximisation of this ratio, the ‘logical entropy’, in the case of a gas in a box (initially static), without engaging the formalism of Statistical Mechanics. We also apply this argument to a model of Neo-Darwinian Evolution, e.g. that of a terrestrial environment which serves as the habitat for multiple populations which do not interbreed. Here, a state of Natural Selection, in which one population is ‘selected’ to surpass the other in number, is seen to be inevitable by our same understanding of probability.

Discussing the relationship between probability and uncertainty, we invoke the Principle of Indifference to give an epistemological treatment of the concept of scale. We establish the meaning of a difference in scale, and then, treating time more explicitly, we assert the stability of deterministic systems. We conclude that the character of Quantum Mechanical laws is a consequence of the impossibility of constructing microstates and macrostates to describe any collection of particles at the smallest scale extant, presenting an interpretation of Quantum Mechanics in which powerful treatments of observation, probability, and Correspondence are all possible.

2 Statistical Physics

Consider a box, divisible through the centre, occupied by a number of particles of a simple, motionless gas. We are free to pick any microstates and macrostates to describe the system, so long as the two are on different scales. In this example, we take a given microstate to be a specification of the position and velocity of every particle of gas and a given macrostate to be one of three possibilities: either there are particles only on the left; or there are particles only on the right; or there is at least one particle on each side. Note that a specification of one microstate determines one macrostate, but not the other way around.[1]

Assume each microstate is accessible with equal probability, and, beginning with an empty box, construct an arbitrary microstate by placing each gas particle in the box sequentially, assigning a random position and zero velocity. The probability of occupying a given macrostate at a particular point in this procedure is understood by examining a tree diagram, (Figure 1), which recognises that a random arrangement of particles is the end of a procedure by which all of the particles are added independently. Each microstate is a ‘possible history’ of the construction of the system, corresponding to a path in such a schematic. (Now a microstate also specifies the order in which the particles are added to the box.)

If we have two particles, then there are twice as many microstates corresponding to the macrostate in which both sides are occupied as there are corresponding to each macrostate in which only one side is taken; with more particles, the tree diagram becomes exponentially larger. For a system comprising stationary gas particles in a box, the macrostate with both sides occupied is the

[1] Macrostates supervene on microstates.

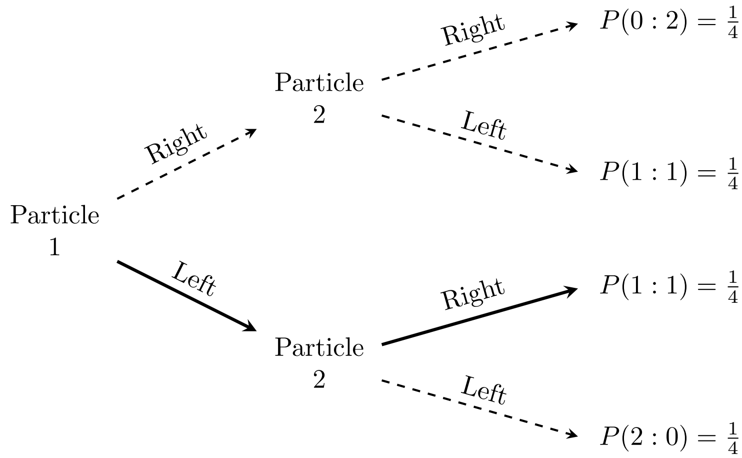

Figure 1: A tree diagram of the construction of a static and binary gas in a box. The solid path through the diagram corresponds to one particular microstate, and the dashed paths represent other possible microstates. Note that dashed lines are a bad metaphor for possibility.

most likely, since it is precisely for this macrostate that the logical entropy is maximised. For any system in which there are many more than two particles, the maximum value of the logical entropy is enormous compared to the alternatives. From the assumption that the microstates and macrostates for a given system are constructed such that the microstates are all equally likely, we predict that the macrostate with the most microstates is the most likely state in which to find the system. Moreover, this preferred macrostate is more likely than the *sum* of the alternative states.

3 Natural Selection

Now take a canonical situation in which Evolution occurs, with Nature selecting between two species, A and B, of equal initial populations, but the former having a phenotype more fertile by a factor of three. Also assume that individuals of both species have a two-thirds chance of dying soon after birth. The development of this system may be partitioned into equally likely possible histories that determine precisely which As and which Bs will have lived and died by the time the macrostate is reckoned. Each of these descriptions will be a microstate. A macrostate, on the other hand, perhaps counts only the ratio of As and Bs alive after each generation. The microstates are the possible histories of the development of the entire ecosystem, and the logical entropy is maximised when Natural Selection selects the more fit and fecund phenotype to be more successful than its competitors, establishing the system in a macrostate with a high ratio of As to Bs.

If a random microstate is constructed, even with such a simple model, the corresponding macrostate will invariably be one in which the As are 'naturally selected' for their fecundity. The single most likely macrostate after one generation, in the above example, predicts four surviving infants of species A for each one surviving infant of species B. A few generations later, and the As will most likely vastly outnumber the Bs, this eventuality corresponding to the Evolution of a higher birth rate in the ancestor of the two species in question. A state for a system in which Evolution of this sort occurs is a state to which the principle of Natural Selection applies.

The result of Evolution, in which there are many more individuals of phenotype A, may be driven by any sort of microdynamics, these the actual physics by which a microstate is chosen. One species may be more fertile or superior in combat. There may be competition for resources. Certainly, in most models, it is an intricate mixture of factors which induces Evolution, but it is just such a set of models which this methodology allows us to treat: in each case, we need only construct the microstates of the system such that each is equally likely. Natural Selection depends not at all on how changes in phenotype arise; it only requires descent with random modification.

The development of a possible history for a terrestrial environment is analogous to the construction of a static gas in a box by the placement of each particle independent of the rest: each column of the tree diagram is descended from the previous column, but with a random modification comprising either a small change in phenotype or the addition of a particle, respectively. Thus, the construction of a thermodynamic system can be understood as obeying Natural Selection, in that a gaseous macrostate with a higher logical entropy is more 'fit' to survive a sequence of random perturbations applied to a previous configuration of that same macrostate; as particles are added to the box, the asymmetric macrostates 'die out', while the remaining possibility persists until it is the only one left.

4 Scale

Many physicists understand that there is a deep connexion between probability and uncertainty. The root of this relation is the Principle of Indifference, which says that for mutually exclusive, jointly exhaustive and indistinguishable possibilities, the probabilities of each being realised are equal in magnitude. The reasoning is as follows: if the possibilities are indistinguishable, then none could be preferred. Microstates, possible paths in state-space, are just such possibilities, and these are the subject of representation by tree diagrams. As in the above example diagram, the elements of the last column are distinct destinations, and the paths by which one arrives at a destination are 'possible histories' of its achievement. Each route is equally likely, and the number of routes which end at a destination is proportional to the probability of arriving there.

This criterion of indistiguishability is a problem, however. If multiple possibilities are indistinguishable, how can one recognise their independent existence?

Rather, the different possibilities are not indistinguishable; it is merely inconceivable that what distinguishes them should affect their relative likelihood. We here arrive at the notion of scale: the features that distinguish possibilities are said to be on a smaller scale than the system itself, so that these features serve only as labels, and each possibility for the system remains equally likely.

In creating a model for a physical system, adopting simplifying assumptions, one assigns both the possibilities and the labels which differentiate them, so a difference in scale is a specification of scope: something modelled is 'small' when one is justified in not considering it, and in not considering something, that thing is not included in the model. Physical size itself is not what is important in the above discussions of the Statistical Mechanics and the principle of Natural Selection; it is merely a convenient mode in which to represent the significance of elements of a physical model.

Indeed, when we say that the particles of an ideal gas, for instance, are small compared to the size of the container, we are saying that, in our model, these particles are point particles. That is, that they have zero extension, and they experience no microdynamics, for point particles are precisely those which have random interactions. (All properties besides position and its derivatives are not included in the model.) These objects with no internal structure, then, are the most basic constituents of *every* system. In a model, there truly are no microdynamics; the macrodynamics are the only extant definitions of state, and the logical entropy of a macrostate is nothing more than its probabilistic weight. In speaking of the choice of microstates and macrostates for a system, we are really discussing different models which may describe the same phenomena.

In our treatments of both the gas in a box and of Natural Selection, we explicitly assumed that the microstates were all equally likely; but now it is clear that this was never needed. The random choice of microstates is implicit in the requirement that the microstates be at a smaller scale than the macrostates, if we extend our conception of scale to involve not only space but also time. Then the transitions among the microstates must be at a small scale, and the microstates themselves are therefore always randomly selected, either in building a static system or dynamically. That is, the timescale of the microdynamics must be significantly less than the period for which the system as a whole is observed. The number of gas particles in the box must also be great, if their physical size is to be indicative of their importance.

With this in mind, we see why the Fundamental Postulate of Thermodynamics did not need to be considered explicitly in our generalisation of the Second Law. If microstates and macrostates are to earn their titles, the Postulate may be indirectly assumed. We can now say something about the gas in a box even if the velocities of the particles are non-zero. In this case, the temporal evolution of the system is determined by the speed of particles and the time interval for which the system is observed. The gaseous system now *evolves* toward a state characterised by an even distribution of particles, as a biological system undergoes Evolution, and the motionless gas is constructed: in each case, the state of evolution or construction is itself the steady (macro)state.

5 Determinism and chaos

The two-body problem, being solvable, is an example of a deterministic system, and it is solvable because it is elementary: it contains no sub-problems, that is, internal systems which are themselves problems on the same scale as the whole. (These are different from microdynamics, which are explicitly on a lower scale than this.) In the case of the n-body problem, a sub-problem may be that same n-body problem constructed in terms of groups of those n bodies. This construction is possible for three or more bodies, but impossible for two: if one of the bodies in the two-body problem is removed, the system is not immediately described; it is then indescribable and in no way a 'problem'. The two-body problem is solvable because it is the *simplest* n-body problem and therefore contains no sub-problems. On the other hand, the three-body problem, for instance, does contain a sub-problem, namely the two-body problem, and so is not deterministic but instead chaotic in behaviour.

Sub-problems lead to chaotic behaviour, because the partitioning of macrostates into microstates must have no structure; it must be a simple list of microstates assigned to macrostates, as in the tree diagrams. With sub-problems, simply counting the proportion of microstates per macrostate is insufficient for the development of a conclusion with regard to the maximisation of the logical entropy, because a tree diagram cannot describe the dynamics of the sub-problems, the existence of which is incompatible with the equal probability of each option in each fork: the interdependence of sub-problems requires that the choice of path not be random. Thus, the existence of sub-problems leads to chaotic behaviour in the three-body problem and the general n-body problem, where small perturbations have global effects (in space and in time). Notably, sub-problems are not the only condition by which determinism may fail, as we shall see. At least, arguments invoking sub-problems are not always the most intuitive ways of demonstrating the chaotic behaviour of a given system.

More can be said about chaotic systems, namely that, where any system is, at any time, in one of an enumerable set of definite states, with a chaotic system specifically, this set has more than one member, and a chaotic system is chaotic as it oscillates unpredictably among the multiple possibilities. With determinism, in contrast, there is only one state to be occupied. Consider a spinning coin in a coin toss, another example of a chaotic system. Here there are two states, 'heads up' and 'heads down', and the coin may indeed be said to occupy one or the other at any given point in the toss; if there were instead just one state, this would provide a stable equilibrium for the coin, as exists for a spinning sphere. For any system, the presence of more than one distinct macrostate is equivalent to the absence of distinct (i.e. random) microstates.

A chaotic system, such as the three-body problem, may be viewed not, first and foremost, as one whose immediate future cannot be determined, but one for which no logical entropy can be defined because one can construct no formalism which directly partitions possible histories into equally likely microstates. If an appropriate logical entropy could be constructed, then there would be a state with the greatest logical entropy, and the future of the system could then be predicted [to be an occupation of that state]. Contrariwise, if a system behaves deterministically, it does so only for lack of chaotic behaviour.

6 Quantum Mechanics

Couple these notions to a simple assumption: that a given system is composed of *elementary particles*, (and that the system itself is on the scale of those particles). An elementary particle here is one that is indivisible, in the sense that when an elementary particle is "divided", it yields constituents that are on the same scale as the original, as is seen with the creation and annihilation of particles in the Standard Model. The immediate result is this: the existence of elementary particles limits the construction of the microstates of small-scale systems, and so a system on such a scale is non-deterministic for want of a viable definition of entropy.

Thus we understand the peculiar form of the laws of Quantum Mechanics to be a logical consequence of the presence of actual elementary particles, the scale of which is the domain of the physical theory. Reintroducing earlier terminology, indistinguishable elementary particles *really are indistinguishable*, for they cannot be marked by (insignificant) labels as can, for example, the two sides of a coin. We remark that, in this view, the Correspondence Principle remains valid, for classical[2] determinism is supposed to be possible only at a scale larger than that of elementary particles. It is not just sufficient that systems be large to behave classically, it is necessary, too.[3] Quantum Mechanics is then not a break with the principles of Classical Mechanics; rather it is the consequence of them, these principles being the very ones which allow a treatment of the classical two-body problem. Determinism is fundamental, and it manifests differently near and far from a minimum scale.

This new perspective on the laws of Quantum Mechanics allows for some valuable insights into, for instance, the nature of observation. In Quantum Mechanics, to observe an elementary particle directly is to learn the dynamics of the constituents of the relevant system and to be able to incorporate them in the model used. These dynamics then cannot relate well-defined microstates, because, to be microdynamics, they must be unknown, by definition. With observation then, the system itself changes, for there no longer *can* be any microdynamics, and determinism no longer applies. Then the dynamics of the observed particles are the only dynamics left to be seen.

This is made clear by the example of the double-slit experiment, in which light is shone through two closely spaced slits and onto a screen. Here, an interference pattern may be observed only when no apparatus is used to determine which slit the photons are passing through. To say that such an observation

[2]By 'classical' we here mean (not Quantum Mechanical); we do not mean (not Relativistic).

[3]Black holes are here excepted, these objects being found to experience both classical and quantum effects, not because their surface gravity is high, but because they really are the "elementary particles of General Relativity". This is so, not because there is nothing smaller than a black hole, but because black holes have no internal structure and corresponding microdynamics.

disturbs the system begs the question, as is known, for why should it disturb the system in this way? That is, why should counting the particles destroy the interference pattern?

When the individuals are observed, the destruction of the interference pattern occurs because the microdynamics among the photons (that is, their diffraction through the slits and their quantal nature) can no longer be ignored in the prediction of the final pattern. It is when the microdynamics of their interactions are no longer random, i.e. no longer unknown, that there no macro*dynamics* (large-scale interference). Ignorance of the particular dynamics is a necessary condition for the probabilistic assumptions of determinism to make sense, and this condition cannot be met when the elementary particles are observed. Further, when the macrodynamics cease to exist, the pattern visible on the screen should be the simple sum of the dynamics of the constituents themselves, as experiment verifies: that is, the pattern visible should *be* the microdynamics, such as they are.

7 Conclusion

With the failing of determinism on the scale of elementary particles, Quantum Mechanical systems behave very similar to the chaotic systems discussed above. As Quantum Mechanics gives a probabilistic result that a sub-atomic particle will be in this or that state, so does a model of a spinning coin yield a probability for each possible state of the coin at any time (these probabilities being decided by the coin's fairness). Observation and the consequent collapse of the wave-function place the Quantum Mechanical particle in one or the other state, as the coin's position function "collapses" to either 'heads up' or 'heads down' when it is observed. In both cases, if the systems are then unobserved for sufficient time, the objects regain their unpredictability. The quantisation of states in Quantum Mechanics is analogous to a multiplication of states in Chaos Theory. The three-body problem too behaves in this fashion, with its oscillations being between states of high and low potential energy.

The only difference between quantum and chaotic phenomena is that, in the former, it is the minimum scale of the system that is the cause of the unpredictable behaviour, rather than the existence of sub-problems, (in the case of three-body problem,) or the spin of the coin, (in the coin toss). Determinism always fails to describe a system because of properties of that system which prohibit the construction of microstates and macrostates in the required fashion.

A probability-based determination of the state of a system is, in some sense, the 'default' determination of all dynamics, for without a reason to prefer a given eventuality, one must acknowledge each possibility as equally likely. Only when a system transcends spatial and temporal scales, may one recognise an eventuality as favoured. For instance, the systems of Natural Selection and Statistical Mechanics transcend scales; but those of Quantum Mechanics and Chaos Theory do not. And having seen this, we should be particularly satisfied to see the latter three are all explained by the same principles. These three disciplines of theoretical physics are precisely those in which probability plays a foundational role. The symmetry is excellent: probability manifests itself only once in the world.

A Appendix: E.T. Jaynes

The author here recognises a number of similarities between his own thought and that of E.T. Jaynes, whose work he first encountered only as this article was nearing completion. The reader is referred to Jaynes's corpus, and specifically the works cited in the bibliography, for the purpose of comparison, and especially if he seek a mathematical treatment of the common notions.

References

[1] E.T. Jaynes. Prior Information in Inference. 1982.

[2] E.T. Jaynes. Macroscopic prediction. In H. Haken, editor, *Complex Systems — Operational Approaches*, page 254. Springer-Verlag, Berlin, 1985.

[3] E.T. Jaynes. How Should We Use Entropy in Economics? doi:10.1.1.41.4606, 1991.

[4] E.T. Jaynes. The Second Law as Physical Fact and Human Inference. doi:10.1.1.77.7388, 1998.