Microstates, Macrostates, Determinism and Chaos*

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Abstract

The Second Law of Thermodynamics and Darwinian Evolution are founded on arguments of identical form, the employment of which is sufficient for drawing conclusions as to the gross behaviour of diverse physical systems. These arguments are a part of the description of any system which may be called deterministic; chaotic systems, in contrast, are unpredictable precisely because they cannot be thus treated. The chaotic behaviour of the general n-body problem, and the non-classical features of Quantum Mechanics, follow from the impossibility of fulfilling the prerequisites of determinism in the presence of 'sub-problems', and at the level of elementary particles, respectively.

1 Introduction

When a system is able to be described simultaneously at two different scales, a probabilistic treatment of the assignment of small-scale states ('microstates') to large-scale states ('macrostates') specifies the system's large-scale behaviour for arbitrarily far in the future. Prediction is this act of specification, and a predictable system is one that is deterministic. A deterministic system will always occupy the macrostate with the greatest number of corresponding microstates—the macrostate's 'logical entropy'—which value is proportional to its probalility of being realised.

2 Statistical Physics

Begin with an empty box divisible into two parts. Place some number of particles in the box consecutively, assigning to each particle a random position and zero speed. Take a given microstate of the system at any point in time to be a specification of the position [and (null) velocity] of every particle of the frozen gas, and a given macrostate to be one of three possibilities: either there are particles only on one side; or there are particles only on the other side; or there is at least one particle on each side.[1]

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[1] Note that macrostates supervene on microstates.

The probability of occupying a given macrostate at some point in this additive procedure is understood by examining a tree diagram which recognises that a random arrangement of particles is the end of a process by which all of the particles are introduced independently (see Figure 1). Each path through the tree diagram is a 'possible history' of the construction of the system.

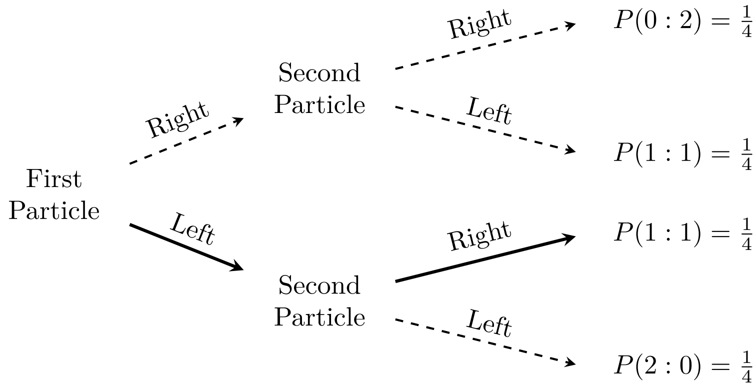

Figure 1: A tree diagram of the construction of a static and binary gas-in-a-box.

In general, the macrostate with both sides occupied is the most likely state for the system to occupy, since it is in this macrostate that its logical entropy is maximised. When the box contains two particles, there are twice as many microstates corresponding to the macrostate in which both sides are occupied as there are corresponding to each macrostate in which only one side is taken. When the box contains many more than two particles, the maximum value of the logical entropy is enormous compared to the alternatives. Indeed, the occupation of the macrostate with the most microstates is both most likely and *very* likely: this state is favoured over even the sum of the alternative states, of which there can only ever be a few.

3 Natural Selection

Consider two species, A and B, of equal, large initial populations. Assume that the As are more fertile by a constant factor. The development of this system may be partitioned into equally likely possible histories that specify precisely which As and which Bs lived and died—into microstates thereby defined. A macrostate, on the other hand, perhaps counts only the ratio of As and Bs alive after each generation. For a random microstate, the corresponding macrostate will probably be one in which the As have been 'naturally selected' for their superior reproductive ability. After many generations, the As will [almost certainly] vastly outnumber the Bs, this eventuality corresponding to the Evolution of a higher birth rate in the common ancestor of these two species.

A state of a system in which Evolution occurs is a state to which the principle of Natural Selection applies, and the state to which that system evolves is the one in which its logical entropy is maximised. Evolution may be driven by any sort of microdynamics, these the actual physics by which a microstate is chosen. One species may be more fertile or superior in combat; there may be competition for resources. It may be an intricate mixture of factors which induces Evolution. The only necessity is that one [be able to] choose the microstates of the system such that each is equally likely. Natural Selection depends not at all on how changes in phenotype arise; it requires only descent with random modification.

The development of a possible history for the action of some Natural Selection is analogous to the construction of the static gas-in-a-box by the placement of each particle independent of the others. In the former case, the random modification of the system is a small change in phenotype; in the latter, it is the addition of a particle to the statistical system. One may construct a tree diagram for the Evolution of individual biologic traits, and one may describe each column of any tree diagram as 'descended' from the previous columns. Indeed, the construction of the static gas-in-a-box may be understood as obeying Natural Selection, with a gaseous macrostate of a higher logical entropy being more 'fit' to have survived the perturbations applied to previous configurations of that same macrostate; as particles are added to the box, the asymmetric macrostates 'die out', while the remaining possibility persists until it is the only one left.

4 Scale

The Principle of Indifference says that for mutually exclusive, jointly exhaustive, and indistinguishable possibilities, the probabilities of each being realised are equal in magnitude. If the possibilities are indistinguishable, then none could be preferred. More precisely, the different possibilities must be indistinguishable in the sense that whatever distinguishes them should not affect their relative likelihood; but they must be able to be identified as distinct. The non-dynamical features ('labels') that do distinguish equally likely possibilities are said to be on a scale smaller than that of the system itself.

In creating a model for a physical system, adopting simplifying assumptions, one assigns both the possibilities and the labels which differentiate them, so a difference in scale is a specification of scope: something modelled is 'small' when one is justified in not considering it, and in not considering something, that thing is not included in whatever model is being proposed. Point particles are precisely those dynamical objects which have random interactions. These objects with no [significant] internal structure, then, are the most basic constituents of every deterministic system.

The Fundamental Postulate of Thermodynamics did not need to be considered explicitly in the above generalisation of the Second Law, because in a dynamical gas-in-a-box (i.e. one in which the velocities of the particles are non-trivial) the temporal evolution of the system is determined by the speed of particles and the time interval for which the system is observed. Such a gaseous system then evolves toward a state characterised by an even distribution of particles, and the state of the evolution toward the state with the maximum logical entropy is itself a (steady) [macro]state with a maximised logical entropy.

5 Determinism and Chaos

The two-body problem, being solvable, is an example of a deterministic system, and it is solvable because it is elementary: it contains no sub-problems—that is, internal systems which are themselves problems on the scale of the system itself. In the case of the n-body problem, a sub-problem may be that same physical system constructed in terms of groups of those n bodies. Such a construction is possible for three or more bodies, but impossible for two: if one of the bodies in the two-body problem is removed, the system is then indescribable (and in no way a 'problem'). The two-body problem is solvable because it is the simplest n-body problem and so contains no sub-problems. On the other hand, the three-body problem, for instance, does contain a sub-problem (namely the two-body problem) and so is not deterministic, but instead chaotic, in behaviour.

Whereas any system is, at any time, in one of an enumerable set of possible states, with a chaotic system specifically this set has more than one member; a chaotic system is chaotic as it oscillates [unpredictably] among the multiple possibilities. With determinism, in contrast, there is only one state [likely] to be occupied. Consider a spinning coin in a coin toss, another chaotic system. Here there are two states, 'heads up' and 'heads down', and the coin may indeed be said to occupy one or the other at any given point in the toss; if there were instead just one state, this would provide a stable equilibrium for the coin, such as exists for a spinning sphere.

Sub-problems involve chaotic behaviour because they are interdependent; with sub-problems, no logical entropy can be defined, because one can construct no formalism which directly partitions possible histories into [equally likely] microstates. Indeed, a tree diagram cannot describe the dynamics of the sub-problems, the existence of which prevents the choice of path through the tree from being random. If and only if an appropriate logical entropy could be constructed would there be a [single] state with the greatest logical entropy, and the future of the system could then be [deterministically] predicted [to be an occupation of that (stable) state].

6 Quantum Mechanics

Now assume that a given system is composed of 'elementary particles' [and that the system itself is on the scale of those particles]. An elementary particle here is one that is indivisible in the sense that, when it is 'divided', it yields constituents that are on the same scale as the original. Elementary particles are indistinguishable because they cannot be marked by labels as can, for example, the two sides of a coin.

The 'peculiar' form of the laws of Quantum Mechanics is a logical consequence of the presence of elementary particles, the scale of which is the domain of the physical theory. To observe an elementary particle [directly] is to learn the dynamics of the absolute smallest constituents of a small-scale system and to be able to incorporate them in the model used. These dynamics then cannot relate well-defined microstates, because, to be microdynamics, they must be unknown. With observation at the smallest scale, then, the system itself changes, for there no longer *can* be any microdynamics at that scale, and the logic of determinism no longer applies. Then the dynamics of the observed particles are the only dynamics left to be seen.[2]

[2] In the case of the double-slit experiment, the determinism prohibited by observation is what produces the large-scale interference pattern visible on the screen when the system is undisturbed.

It is not just sufficient that systems be large to behave classically; it is necessary, too.[3] Quantum Mechanics is then not a break with the principles of Classical Mechanics; rather it is the consequence of them, these principles being the very ones which allow for the classical treatment of, e.g., the two-body problem. Determinism is fundamental, and it manifests differently at and away from the minimum scale.

7 Conclusion

As Quantum Mechanics gives a probabilistic result that a subatomic particle will be in this or that state, so does a model of a spinning coin yield a probability for each possible state of the coin at any time (these probabilities being decided by the coin's fairness). In Quantum Mechanics, the collapse of a wave-function places the system in one or the other state, as a coin's position function 'collapses' to either 'heads up' or 'heads down' when it is observed. In both cases, if the system then goes unobserved for sufficient time, the object regains its unpredictability. The three-body problem, too, behaves in this fashion, with its oscillations being between states of high and low potential energy. The quantisation of states in Quantum Mechanics is part of the multiplication of states in chaos; precisely with determinism is there ever only one possible smooth end-state.

The difference between quantum and classical chaos is that in the former it is the minimum scale of the system that is the cause of the unpredictable behaviour, while in the latter it is the existence of sub-problems (as in the three-body problem) or the flatness of the coin (as in the coin toss). Determinism always fails to describe a system because of properties of that system which prohibit the construction of microstates and macrostates in the required fashion. A probability-based determination of the state of a system is, in this sense, the default determination of all dynamics. Only when a system transcends spatial and temporal scales may one recognise one possibility as favoured. For instance, the systems of Natural Selection and Statistical Mechanics transcend scales; but those of Quantum Mechanics and Chaos Theory do not. Notably, these last three disciplines are just those subjects in theoretical physics in which probability plays a foundational role.

Appendix A E.T. Jaynes

The author here recognises a number of similarities between his own thought and that of E.T. Jaynes, whose work he first encountered only as this article was nearing completion. The reader is referred to Jaynes's corpus, and specifically to the works cited in the bibliography, for the purpose of comparison and especially if he seeks a mathematical treatment of the common notions.

References

[1] E.T. Jaynes. Prior Information in Inference. 1982.

[3]Black holes are exceptions: like the elementary particles of the smallest scale, they have no internal structure and so, too, no microdynamics.

[2] E.T. Jaynes. Macroscopic prediction. In H. Haken, editor, *Complex Systems — Operational Approaches*, page 254. Springer-Verlag, Berlin, 1985.

[3] E.T. Jaynes. How Should We Use Entropy in Economics? doi:10.1.1.41.4606, 1991.

[4] E.T. Jaynes. The Second Law as Physical Fact and Human Inference. doi:10.1.1.77.7388, 1998.