

INERTIAL FORCES AS THE INTERNAL MOTION OF EXTENDED BODIES

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Abstract

I re-examine inertial frames of reference in connection with the concept of motion embedded in Newton's laws, distinguishing between the translational motion of point masses and the internal motion of extended bodies. Applying Newton's Corollary VI to the parts of a single body, I show that a body can register its acceleration only through internal motion, specifically through tension and compression, or through the relative motion of unconstrained parts on which the accelerating forces do not directly act. In an accelerating frame, inertial forces are given by the readings of instruments at rest in that frame; the classic thought experiments that purport to demonstrate the existence of inertial forces in fact exhibit precisely those instrument readings. A frame of reference is therefore identifiable as inertial simply by what the instruments at rest in it register.

Keywords

philosophy of physics, classical mechanics, motion, frames of reference,
inertial forces, thought experiments

1 INTRODUCTION

The basic laws of mechanics are widely understood to be valid only in inertial reference frames (Møller [1952] 1955, 4; Friedman [1983] 2014, 13). In accelerating, non-inertial reference frames, models of mechanics require the inclusion of ‘fictitious’ inertial forces (or ‘pseudo-forces’) in order to describe correctly the behavior of bodies (Feynman, Leighton, and Sands [1963] 2010, §12-5; Sciama 1969, 7). Yet there is debate over the very meaning of the ‘motion of a reference frame’, which seems to require some notion of absolute space (DiSalle 1990, 139–40).

These issues are especially relevant in light of work by Saunders (2013) showing that the concept of an inertial frame is not in fact integral to Newtonian mechanics. Saunders develops a space-time representation of Newton’s laws that encodes only the precise dynamical symmetries of the classical theory, taking into account Corollary VI to the Laws of Motion, and demonstrates that linear acceleration, unlike rotation, is undetectable when it is shared equally by all bodies in a system. I would like to add to this discussion by asking what physical states of a body make its acceleration and rotation measurable, and thus what picks out a frame of reference as inertial. To do so, I re-examine the classic thought experiments invoked to demonstrate inertial forces, beginning with an old distinction between two kinds of motion (cf. Aristotle 1984, IV.5; Copernicus [1543] 1978, I.8): 1) motion that is a *change of position* of a body, and 2) motion that is *internal to that body* (for example, rotation).

The thought experiments historically used to demonstrate the existence of inertial forces in accelerating frames of reference are interpreted in several ways. In one interpretation, a passenger in an accelerating train feels an inertial force (Born 1962, 77),

which “arises purely from the choice of a non-inertial frame of reference” (Sciama 1969, 7). In Newton’s thought experiment of the rotating spheres, he took the tension in the string connecting the spheres to be an effect by which true circular motion is distinguished from relative motion ([1726] 1999, 412, 414), while Mach regarded the analogous deformation in Newton’s bucket as an effect of rotation relative to the distant stars (Mach [1883] 1919, 232). Others posit a real force alongside the fictitious one (Roche 2001, 402) or take inertial forces themselves to be real (Lanczos [1949] 1986, ch. IV; Noll 2007, 3).

In mathematical physics, ‘inertial force’ is used both for the ‘fictitious’ force added to make the Second Law hold in an accelerating frame and for what Newton called the *vis inertiae*, a body’s resistance to acceleration. I will reserve the term ‘inertial force’ for the fictitious force alone. Whereas an inertial force in a given frame is assigned to all bodies or point particles described within it (whether accelerated or not), and it has no physical source or third-law partner, the *vis inertiae* is exercised only by an accelerated body, as when a passenger pushes back against their seat or as each of Newton’s spheres pulls outward on the string. Because a body held at rest in an accelerating frame is accelerated by whatever holds it there, the inertial force assigned to it in that frame equals the resistive push or pull that it exerts (Proposition 1). When used to demonstrate inertial forces, the classic thought experiments invite a conflation of the two by describing each body in its own rest frame—the passenger in the frame of the train, and the spheres in a frame co-rotating with them. In each such frame, the fictitious inertial force and the body’s real resistance share the same magnitude and direction. In both of these experiments, what the passenger feels and what Newton’s string exhibits are simply the internal *compression* and *tension* through which an extended body

registers its acceleration.

The requirement that the body be extended follows from Newton's Corollary VI to the Laws of Motion, according to which bodies urged along parallel lines by equal accelerative forces move among themselves just as if no such forces acted ([1726] 1999, 423). Saunders (2013, 34–39) applies Corollary VI to systems of bodies to show what their relative motions cannot reveal, and Wallace (2020, §4) gives the conditions under which the internal dynamics of a subsystem is insensitive to the subsystem's acceleration. I apply the Corollary to the constituent parts of a single extended body to see how its acceleration becomes detectable. Because acceleration impressed equally on all of a body's constituent point particles leaves their relative motions unchanged, the body can register its acceleration only through the unevenness with which the accelerating forces act across its parts, such as the compression of the passenger's back against the seat or the tension in the string. Indeed, gravity alone imparts the same acceleration to every body at a given position, which is why general relativity does not treat it as a force. The inertial frames at issue here are, accordingly, those that fall freely, do not rotate, and remain rigid (§3).

Instead of treating a frame of reference as an abstract system of coördinates, I treat a frame of reference itself as a composite body whose parts are observers. When such a frame accelerates and its parts are held in position by the push and pull of their neighbors, it undergoes compression or tension like the seated passenger, while an unconstrained part released within it drifts freely, like a steward on roller skates in the aisle. Under rotation, gyroscopes at rest in the frame precess relative to it, and unless each of its parts falls freely, the frame is stretched like Newton's string; under expansion or shear, the mutual distances between its parts change. A frame's inertial forces are

thus given by these internal readings and assigned to all bodies or point particles described within it (Jantzen, Carini, and Bini 1992), and a frame is inertial precisely when all such readings vanish, in which case its parts may be described without loss as independent *point particles in translation*. What picks out a frame as inertial, then, is what it registers internally. This is the physicists' operational definition of an inertial frame, via zero-reading accelerometers and gyroscopes (cf. Misner, Thorne, and Wheeler 1973, §13.6). I assume the universality of free fall, which ensures that instruments of every composition pick out the same frames.

I am not proposing any alternative to, or extension of, Newtonian mechanics. Using nothing beyond Corollary VI, proper acceleration, the divergence of stress, and the Third Law, I show that the accelerometers and gyroscopes by which one defines an inertial frame register acceleration and rotation only through their internal motion; replacing an extended instrument with a single structureless point idealizes away the internal degrees of freedom by which it registers them.

Section 2 distinguishes the two kinds of motion and treats frames of reference as extended bodies. Section 3 distinguishes pushing a body from accelerating it, and defines the *vis inertiae* and the inertial force. Section 4 applies these distinctions to the thought experiments, and section 5 asks what makes acceleration and rotation measurable, and what, in consequence, picks out a frame as inertial.

2 TRANSLATIONAL AND INTERNAL MOTION

Definition 1. A **point particle**, or **point mass**, is an object occupying a single point in space at any given time, with a mass, m , and no parts. It has no extension, shape, or

orientation.

Definition 2. An **extended** or **composite body** of mass m is composed of point particles, its parts, in some definite arrangement. Unlike a point particle, it has extension, shape, and orientation in space.

Definition 3. **Translation** is the motion of a point particle through space. The translation of an extended body is that of its center of mass.

Definition 4. **Internal motion** is a change in the size, shape, or orientation of an extended body, such as rotation, compression, or expansion, including static deformation. Size and shape are measured from a reference configuration of the body, and orientation against a standard of rotation, such as a gyroscope.

When one says that an object is moving, one may mean that it is moving translationally, like an object riding on a conveyor belt, or that it is moving ‘internally’, like a rotating body or a working machine. In the former case, the object is described as a point particle, with no parts and no internal motion. In the latter, the body is treated implicitly as comprising multiple parts in some arrangement, such as the sides of a spinning sphere or the gears of a clock. Just as it is meaningless to speak of the internal motion or rotation of a point particle, the translational motion of an extended body—even its position—is undefined without some convention, such as the center of mass in Definition 3.

Either kind of motion can be modeled in terms of the other. We may describe a spinning ball, for instance, either in terms of the rotation of the extended ball itself, or in terms of the translational motion of each of its *parts* treated *as point particles*, each traveling in a circle around the axis of rotation. Conversely, the translation of an object

along a conveyor belt may be described as the internal motion of an extended assembly line comprising both object and belt, where the assembly line as a whole remains at rest (cf. Husserl [1934] 1981, 227) and only its internal arrangement changes. The difference lies in whether we treat the parts of a system as belonging to a single extended body or as independent point particles.

A frame of reference may be treated in the same way—as a composite body whose parts are the observers at rest in it (§1). Einstein pictured even an arbitrary system of coördinates as a body of this kind: a ‘reference-mollusc’ that can change its form as it moves, with a clock fixed at each of its points ([1917] 1920, ch. 28). Brown’s dynamical approach similarly treats rods and clocks as material bodies whose behavior the dynamics must explain (Brown 2005). The motion of a body relative to the frame can then be treated, like that of the object on the conveyor belt, as the internal motion of a larger body comprising both; and a frame can have internal motion of its own as its parts change their relative positions. As with the assembly line, whether this relative motion counts as internal to the frame depends on how we group its parts, whereas the readings of the instruments at rest in the frame do not. In this view, a body’s rest frame is simply the body itself, extended by whatever further parts remain at rest with respect to it.

In the mechanics of point particles, all motion is represented as the translational motion of point masses through space; what might be described as an extended body is always represented as a mere collection of point particles in translation (Boltzmann 1974, 143, 224; Hertz [1894] 1899, 126). Thus Newton held that “the motion of a whole is the same as the sum of the motions of the parts; that is, the change in position of a whole from its place is the same as the sum of the changes in position of its parts from

their places” ([1726] 1999, 409). Likewise, the rotation of an extended body is understood as the *circular* motion of its parts as point particles, that is, as the translation of those point particles, with the direction of their motion constantly changing (Galilei [1590] 1960, 64; Berkeley [1721] 1992, §61).

3 FORCE AND MOTION

Corresponding to these two kinds of motion are two distinct effects of a force: it *pushes* or *pulls* on the parts of the body where it acts, and, insofar as that pushing or pulling is unbalanced, it *accelerates* the body as a whole. Pushing a hockey puck across the ice does both, whereas squeezing a spring between one’s fingers compresses it without accelerating it. Point-particle mechanics captures only the net acceleration, defining a force by the translational acceleration it tends to produce and setting the *sum* of the forces on a body equal to its mass times the acceleration of its center of mass (cf. Newton [1726] 1999, 416), so that any unaccelerated body is treated as subject to balanced forces. Yet the puck and the spring are both compressed; they differ in how that compression is distributed across their parts. Whereas the spring is compressed evenly from end to end, the puck is compressed most where it is pushed and not at all at its far side, since each of its parts is accelerated only by the push of the parts behind it.

This difference between the puck and the spring is a direct consequence of applying Corollary VI to the parts of a body (§1). Because an acceleration impressed equally on every part of a body leaves the relative motions of its parts unchanged, a body can register its acceleration, in the sense that some internal state of the body depends on it, only through the compression or tension by which its parts transmit forces to their

neighbors, or through the relative lag of an unconstrained part on which the accelerating forces do not act.

Gravity is unique in acting alike on all bodies. As Newton’s pendulum experiments show, gravity imparts the same acceleration to every body at a given position ([1726] 1999, Bk. III, Prop. 6), which together with Corollary VI—a combination Saunders (2013, 36) calls Corollary VI*—implies that no body can register gravity where it is uniform across the body. Any other force, by contrast, is proportional to the mass of a body only by accident of its composition; since charge-to-mass ratios vary, an electric field that accelerates one body uniformly will still be registered by a body of another composition (38). This motivates not counting gravity as a force at all, so that a body in free fall is subject to no force and a system moving *as a unit* in free fall needs no inertial forces in a model in which it is at rest. Accordingly, with Knox (2014), Newton–Cartan theory (cf. Malament 2012, ch. 4), and general relativity, I count as inertial any frame of reference that falls freely, does not rotate, and remains rigid (§5.2). In such a frame the Second Law reads

$$m\mathbf{a} = \mathbf{F}, \tag{1}$$

where $\mathbf{F}$ is the net force on the body, gravity excluded.

Definition 5. The **proper acceleration** of a point particle or body is its acceleration $\mathbf{a}$ in an inertial frame.

Proper acceleration is what an accelerometer carried by the body reads (Proposition 2), and, like proper time, which is what a co-moving clock reads, it is the same in every frame of reference. Hereafter, ‘acceleration’ means proper acceleration, unless a frame is named. A body resting on the ground, for instance, has an upward

proper acceleration of magnitude g because the ground pushes upward on it and holds it out of free fall, whereas a body in free fall has zero proper acceleration.

Definition 6. The *vis inertiae* of a point particle or body is its resistance to proper acceleration (cf. Newton [1726] 1999, 404–5), not a force on it. While it has a proper acceleration $\mathbf{a}$, it pushes or pulls on whatever accelerates it with a total force of $-m\mathbf{a}$, its *impetus*.

The impetus is an ordinary force, exerted on another body, and by the Third Law and (1) it is equal and opposite to the net force on the body. A passenger accelerated by a seat pushes back against it, and a body resting on the ground pushes downward on the ground with the force that a scale beneath it reads as its weight. A body in free fall exercises no *vis inertiae* at all.

In an extended body, the resistance is exercised part by part. The forces that accelerate a body generally push or pull only some of its parts, and the rest are accelerated by the push or pull of their neighbors, on which each exercises its own *vis inertiae* in return. In an inertial frame, momentum balance for a continuous body reads

$$\rho \mathbf{a} = \nabla \cdot \mathbf{T} + \mathbf{b}, \quad (2)$$

where ρ is the density, $\mathbf{a}$ the proper acceleration of each part, $\mathbf{T}$ the stress tensor, and $\mathbf{b}$ any body force, such as an electric field exerts on a charged body. Where the forces act only at the surface, as when a seat pushes a passenger, $\mathbf{b} = \mathbf{0}$, and each part is accelerated by the net push and pull of its neighbors, $\nabla \cdot \mathbf{T}$ per unit volume. Because squeezing the spring adds nothing to $\nabla \cdot \mathbf{T}$, stress by itself does not indicate acceleration; what distinguishes an accelerating body from a merely compressed one is

a non-vanishing divergence $\nabla \cdot \mathbf{T}$, which as a spatial derivative requires an extended body. Conversely, a uniform body force proportional to mass ($\mathbf{b} = \rho \mathbf{k}$ for a uniform $\mathbf{k}$) accelerates every part alike without producing stress, so that, by Corollary VI, the body registers nothing, although the same field would be registered by an instrument whose parts differ in composition, such as a charged proof mass tethered to an uncharged housing. An extended body therefore registers its acceleration in its stress exactly insofar as the forces that accelerate it are not impressed equally on its parts in proportion to their masses.

An inertial force, by contrast, is assigned to a body by a frame of reference. In the frame of an accelerating train, a seated passenger is at rest, although the seat pushes them forward.

Definition 7. The **inertial force** on a point particle or body in a frame of reference K is whatever must be added to the forces on it for the Second Law to hold in K :

$$\mathbf{F}_{\text{in}} := m\mathbf{a}_K - \mathbf{F}, \quad (3)$$

where $\mathbf{a}_K$ is its acceleration relative to K .

Such a force is called fictitious because nothing exerts it. In a frame that does not fall freely, the inertial force includes what is usually called gravity; in the frame of the ground, for instance, a body at rest is assigned a downward inertial force, $m\mathbf{g}$, which balances the upward push of the ground.

To see how this fictitious force connects to the *vis inertiae*, we can again treat the frame K , as in §2, as a composite body. Let K be rigid, let the part of K at the position of a point particle or element have proper acceleration $\mathbf{A}$, and let K rotate with angular velocity $\boldsymbol{\Omega}$

relative to gyroscopes.

Proposition 1 (Inertial force). *The inertial force on a point particle of mass m , or on an infinitesimal element of an extended body, moving with velocity $\mathbf{v}_K$ relative to K is*

$$\mathbf{F}_{\text{in}} = -m\mathbf{A} - 2m\boldsymbol{\Omega} \times \mathbf{v}_K. \quad (4)$$

It equals the impetus that the particle or element would exercise if it were held unaccelerated in K , and, if it remains at rest in K , the impetus that it does in fact exercise.

Proof. The acceleration of the particle or element is that of the part of K at its position, plus its acceleration relative to K and the Coriolis term: $\mathbf{a} = \mathbf{a}_K + \mathbf{A} + 2\boldsymbol{\Omega} \times \mathbf{v}_K$. By (1) and (3), $\mathbf{F}_{\text{in}} = m\mathbf{a}_K - m\mathbf{a} = -m\mathbf{A} - 2m\boldsymbol{\Omega} \times \mathbf{v}_K$. If a further force holds it unaccelerated in K , then $\mathbf{a}_K = \mathbf{0}$, and by Definition 6 its impetus is $-m\mathbf{a} = \mathbf{F}_{\text{in}}$. If it remains at rest in K , the forces that act on it hold it unaccelerated in K . $\square$

The first term of (4) is $-m$ times what an accelerometer at rest in K at the position of the particle or element would read, and the second is fixed by what gyroscopes at rest in K would read (§5.1). In a frame whose origin accelerates and whose axes rotate, the first term comprises the familiar translational, Euler, and centrifugal forces, and the second term is the Coriolis force.

Although the inertial force and the *vis inertiae* are distinct, the inertial force equals the impetus whenever a body remains at rest in a frame, so in the body's own rest frame its impetus appears as an inertial force acting on it; for any other body, the inertial force equals the impetus the body would exert if held unaccelerated in the frame. This is d'Alembert's principle read in reverse: adding $-m\mathbf{a}$ to the forces on a body amounts to

describing it in a frame in which it is unaccelerated (d'Alembert 1743; Lanczos [1949] 1986, ch. IV).

4 THOUGHT EXPERIMENTS

Consider a train accelerating from a stop, with a passenger sitting in a seat facing forward and a steward on roller skates standing in the aisle (Figure 1). As the train accelerates forward, the steward rolls backward in the train and the passenger is compressed by the seat. Although this setup is often used to illustrate inertial forces in non-inertial reference frames (Born 1962, 77), what the passenger actually feels is a reading, the internal compression of their body, rather than a force pushing them backward. Relative to the train, the steward accelerates backward as an uncompressed unit, behaving like a point particle in translation, whereas the seated passenger remains at rest as a whole and registers the acceleration entirely through the internal compression of their body.

In the frame of the train, the passenger is not accelerating, so the forward push of the seat must be balanced by a backward inertial force. In the frame of the platform, which for horizontal motions is inertial, there is no inertial force, and the push of the seat accelerates the passenger with the train. What is present in both frames is the passenger's *vis inertiae*, exercised as a backward push on the seat and registered in the compression of their body, which is greatest at their back and vanishes at their front. That push appears as an inertial force on the passenger because the thought experiment is described in the frame of the train, the passenger's rest frame. By Proposition 1, the two are equal. The steward, on the other hand, accelerates backward in the frame of the

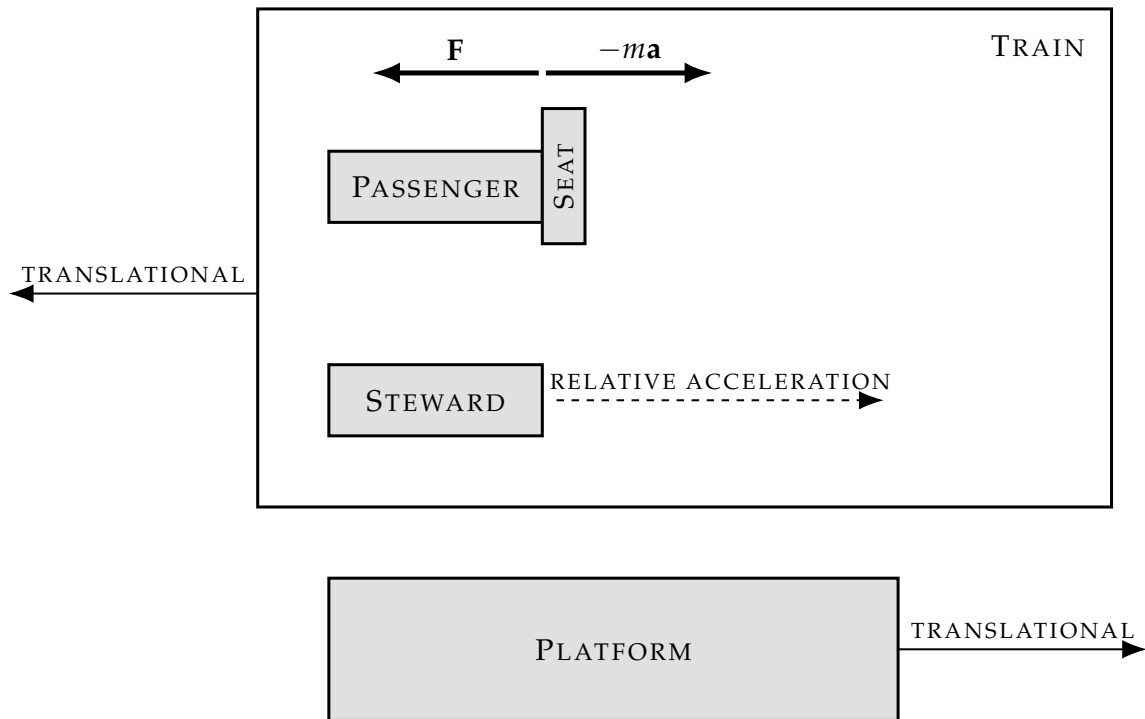


Figure 1: The seat pushes the passenger forward (F), and the passenger, exercising their *vis inertiae*, pushes back on the seat ($-ma$), and registers their acceleration in the compression of their body. The steward, rolling freely, exercises none. Only horizontal forces are shown. The arrows marked 'translational' show the train's translation relative to the platform and the platform's translation relative to the train.

train in the same way as the platform outside it, and neither exercises any *vis inertiae* against the train. The inertial force assigned to each equals, per unit of mass, the passenger's impetus: a push that either would exert only if held at rest in the train. Because this inertial force arises from the acceleration of the train rather than from any interacting body, it has no third-law partner. Finally the passenger, when modeled as a single point mass in the frame of the platform, is subject to no inertial force, and nothing internal to the passenger can register their acceleration; even if the seat is included in

the model along with the passenger's push against it, a point mass has no parts to be compressed.

Rotation might seem to be a special case. The classic argument that true rotation can be known by its effects is Newton's thought experiment ([1726] 1999, 414) of two spheres connected by a string and rotating about their common center of mass (Figure 2). The tension in the string pulls each sphere toward the center, and, by the Third Law, each sphere pulls outward on the string. This outward pull, sometimes called the 'reactive' centrifugal force, is not the inertial centrifugal force, which is assigned to the spheres in a rotating frame and has no third-law partner. It is the impetus by which each sphere exercises its *vis inertiae* on the string. From the observation of this tension, Newton argued, we can compute the true circular motion of the spheres.

Passing to a frame that co-rotates with the spheres both introduces inertial forces for all bodies or point particles described in it and leaves the spheres (and their parts, which as point masses were in circular translation) at rest. For a sphere of mass m revolving with angular velocity Ω at a distance r from the axis, the radial force balance is $0 = -T + m\Omega^2 r$, where T is the tension; here we introduce the centrifugal force to balance the tension in the extended string while still treating the spheres as point particles (e.g. Newburgh 2007, 427–28). If the spheres are treated as extended bodies, each part of each sphere is pulled toward the axis by the parts next to it, and resists with its *vis inertiae*, in whatever frame the sphere is described. By Proposition 1, the centrifugal force on each part then equals the impetus that it exercises on the parts next to it, and the tension in which the spheres register their acceleration appears as what the centrifugal force balances. A free particle described in this co-rotating frame is assigned the same centrifugal force per unit mass as the part of the rotating body at its position,

together with a Coriolis force, the two summing to the impetus that the particle would exert if it were held unaccelerated in the frame. Newton's bucket works in the same way, as the concave surface of the spinning water is a deformation maintained by the internal pressure gradient that imparts to each fluid element both its upward proper acceleration out of free fall and its inward centripetal acceleration toward the axis.

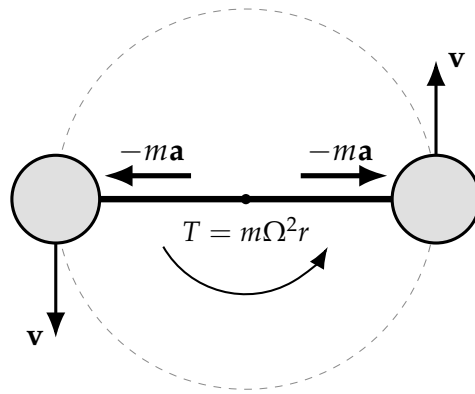


Figure 2: Spheres joined by a string each exercise their *vis inertiae* as a pull on the string ($-m\mathbf{a}$).

5 READINGS AND INERTIAL FRAMES

5.1 ACCELEROMETERS AND GYROSCOPES

For a physical body to measure any quantity, some internal state of the body, such as the extension of a spring or the angle between a gyroscope's rotor and its housing, must depend on that quantity, which is why a structureless point particle cannot itself measure anything. Corollary VI entails, moreover, that a body can register its acceleration only through the unevenness of the forces that accelerate it (§3).

Model an accelerometer as a proof mass m on which nothing acts except a spring of stiffness k that tethers it to a housing, and let the housing remain non-rotating with uniform proper acceleration $\mathbf{a}_h$ across the instrument.

Proposition 2 (Accelerometers). *(i) In steady state, when the proof mass moves with the housing, the extension of the spring is $\Delta = -m\mathbf{a}_h/k$, which is the impetus of the proof mass divided by k . (ii) If the proof mass is untethered and shielded from every force, as in a drag-free satellite, its displacement δ relative to the housing obeys $\ddot{\delta} = -\mathbf{a}_h$. In free fall, $\Delta = \mathbf{0}$ and $\ddot{\delta} = \mathbf{0}$.*

Proof. Work in an inertial frame. (i) The only force on the proof mass is the spring's, $-k\Delta$, and the proof mass has the acceleration of the housing, so by (1), $m\mathbf{a}_h = -k\Delta$. (ii) The proof mass is unaccelerated, so $\ddot{\delta} = \mathbf{0} - \mathbf{a}_h$. $\square$

This is the textbook accelerometer equation, and navigation engineers call the proper acceleration that it measures the 'specific force' (Titterton and Weston 2004, 153–55). The seated passenger acts as a tethered accelerometer in which their own body serves as the spring, while the steward and the train together form an untethered accelerometer whose reading is the steward's drift relative to the car (§4). In both cases, what is measured is the displacement of one part of a composite system relative to another.

Rotation is measured in the same way, by a gyroscope, whose reading is the angle between its rotor and its housing. Parts at rest in a frame that rotates uniformly register its rotation only through the centrifugal part of the first term of (4), which does not give the sense of the rotation, and which vanishes from their readings where gravity supplies their centripetal accelerations, as it can inside matter (§5.2). The Coriolis term is registered only by parts that move relative to the frame, such as a gyroscope's spinning

rotor or a free part drifting from its position. Only an instrument whose parts move relative to its housing, then, registers a rotation in every case.

The point-particle approximation discards all such readings. A single point particle's only states, position and velocity, depend on the frame, and it has no parts whose relative configuration could change. As an elastic body shrinks toward a point, the displacements in which it registers its acceleration shrink with it. An elastic body of size L , density ρ , and elastic modulus E , with proper acceleration a , carries strains of order $\rho a L / E$, so that its reading is a displacement $\Delta \sim \rho a L^2 / E$; the accelerometer of Proposition 2 obeys the same law, since its proof mass scales as ρL^3 and its stiffness as EL . For a given material, design, quality factor, temperature, and measuring time, the ratio of signal to thermal noise falls as L^2 (Gabrielson 1993), dropping below any fixed threshold once the body is small enough.

5.2 INERTIAL FRAMES

We can now say what it is for a frame of reference, taken as a composite body (§2), to fall freely, not rotate, and remain rigid (§3). Let the parts of a frame have proper acceleration $\mathbf{A}$ and velocities $\mathbf{v}(\mathbf{x}, t)$ relative to a local inertial frame. The gradient of the velocities decomposes as

$$\partial_j v_i = \frac{1}{3} \theta \delta_{ij} + \sigma_{ij} + \omega_{ij}, \quad (5)$$

where the expansion θ , the shear σ_{ij} (symmetric and trace-free), and the vorticity ω_{ij} (antisymmetric) are the rates at which the frame changes its size, shape, and orientation (Batchelor 1967, 79–84).

The frame is inertial if and only if its parts have no proper acceleration, $\mathbf{A} = \mathbf{0}$, so that

accelerometers of every composition at rest in it read zero; no vorticity, $\omega = 0$, so that gyroscopes at rest in it do not precess relative to it (cf. Misner, Thorne, and Wheeler 1973, §13.6); and no expansion or shear, $\theta = \sigma = 0$, so that bodies at rest in it keep their distances. Each of these three conditions corresponds to a zero instrument reading.¹ This provides an operational characterization of inertial frames rather than a definition independent of the laws of mechanics, since those laws are what establish the correspondence between the readings and the motion of the frame. The conditions on θ , σ , and ω say that the frame is rigid and non-rotating, and the condition on $\mathbf{A}$ says that nothing is needed to hold its parts in position. An inertial frame is thus a composite body free of internal motion whose parts carry zero-reading accelerometers ($\mathbf{A} = \mathbf{0}$), and may therefore be described without loss as independent point particles in translation.

These are also the quantities that give a frame's inertial forces. By Proposition 1, the inertial forces of a rigid frame are given by $\mathbf{A}$ and the frame's vorticity; in a frame that expands or shears, a term in θ and σ is added, which depends on the body's velocity and on how acceleration relative to a non-rigid frame is defined. This is how inertial forces are defined covariantly, relative to an arbitrary 'family of observers'—a frame in the present sense (Jantzen, Carini, and Bini 1992). A frame is inertial precisely when it

1. The first condition alone does not suffice, because the parts of a frame can all fall freely and yet rotate. In Newtonian gravity, a rigid three-dimensional frame can do so only inside matter of density $\Omega^2/(2\pi G)$, since in a vacuum the tidal tensor is trace-free and the centrifugal tensor is not, so the tides cannot supply the centripetal accelerations of its parts. In a vacuum, then, such a frame registers its rotation in the proper accelerations of its parts, and the condition on vorticity is redundant except for a frame too small to resolve those accelerations. The third condition excludes frames whose parts fall freely without rotating but expand or shear, such as a cloud of particles falling freely through a tidal field.

assigns no inertial force to any body or point particle, which occurs if and only if all of these internal readings vanish.

The characterization of inertial frames by instrument readings does not presuppose which frames are inertial, as their definition through the uniform motion of force-free bodies does, since such bodies are known to be force-free only by their uniform motion (cf. DiSalle 2020, §1.5). Moreover, one can check whether an accelerometer registers an acceleration without knowing its reference configuration (Definition 4), since it registers no acceleration along its axis if and only if turning it end for end leaves its reading unchanged (cf. Titterton and Weston 2004, 223–26). One can likewise check that nothing acts on its proof mass except its spring by comparing it with accelerometers of other compositions at rest with respect to it. Because every force (excluding gravity) acts differently on bodies of different composition (§3) whereas free fall is universal, such instruments agree only when nothing beyond their springs acts on their proof masses. This is why the first condition requires zero readings from accelerometers of every composition: an instrument whose parts all carry the same charge per unit mass reads zero in a uniform electric field, while an uncharged accelerometer at rest with respect to it does not.

Tides are the unevenness of gravity across a body, and so, by Corollary VI, the only aspect of gravity that a body can register. Tides appear both as the relative acceleration of neighboring point particles in free fall and as forces of tension and compression acting on extended bodies (Knox 2014), and a freely falling frame cannot remain rigid where they act. Inertial frames in this sense exist only where tides vanish, and so in general only approximately and locally.

6 CONCLUSION

In three of the classic thought experiments invoked to demonstrate the existence of inertial forces, those putative inertial forces are given by the internal compression, tension, or relative displacement of extended bodies. The bodies held at rest in the frame register their acceleration through these internal deformations, and in that frame, what they register is assigned as an inertial force to all bodies or point particles described within it. Instead of attempting to classify reference frames as inertial or non-inertial by their motion relative to something (cf. Earman 1989, 61–62), we are better served by recognizing that a frame is picked out as inertial by what it registers internally.

This bears on the debate with which we began: the frames that Saunders allows, which do not rotate but may have any linear acceleration, cannot be distinguished by the motions of bodies among themselves, and the conditions that Wallace gives are those under which no instrument within a subsystem registers the subsystem's acceleration. Taken as composite bodies, however, the frames can be distinguished by what accelerometers at rest in them read, and those in which accelerometers of every composition read zero are the freely falling frames that Knox counts as inertial. Saunders himself realizes a non-rotating frame by gyroscopes in orbit, screened from any non-gravitational force that might produce a torque (Saunders 2013, 38), and Knox's condition that all bodies pick out the same inertial frames (Knox 2013, 348) becomes, for instruments, the universality of free fall. On either account, then, the frames are found by what instruments register in their internal motion.

Although a point-particle model can represent such readings as the relative motion of many points, idealizing an extended body as a single structureless point preserves its

center-of-mass trajectory at the cost of removing the internal degrees of freedom that are necessary to register its acceleration. What the classic thought experiments exhibit as inertial forces is, in this sense, the internal motion of extended bodies.

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